Quantum simulations based on measurements and feedback control

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We propose a scheme for performing quantum simulations with atoms in cavities based on a photon detection feedback loop that requires only linear optical elements. Atoms can be stored individually without the need of directly interacting with one another. The scheme is able to simulate any time evolution that can be written as a sum of two-qubit Hamiltonians, .e.g., any next neighbor interaction on a lattice. It can also be made robust against photon losses.

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The concept of building a quantum simulator goes back to the early days of quantum information theory, where Feynman [1] formulated the idea to build one special quantum device to simulate another one. This idea was forced by the impossibility to simulate any growing quantum system efficiently on a classical device because of the exponentially growth of the number of involved parameters. However, this restriction can be ruled out by any quantum device due to the same exponentially behavior. The remaining problem is to find a well controlled quantum system which can be forced to evolve reliably under different time evolutions, especially under the physically mostly relevant next neighbor Hamiltonians.

Several systems have been theoretically proposed as candidates for quantum simulation and enormous experimental effort has been put in the realization of the latter. Atoms are one of the most promising candidates for storing a qubit, the basic quantum information unit for the quantum simulator. Trapped in optical lattices [2] or arranged in ions traps [3, 4] they allow for controllable next neighbor interactions that enable the simulation of e.g., the Hubbard-model [5] or spin-lattices [6].

Quantum simulations are closely related to the generation of entanglement since it is the presence of entanglement that prevents the system to be simulated by classical algorithms. Generation of entanglement between e.g. two atoms is usually achieved by either using direct interactions between the atoms, or the interaction can be achieved indirectly by the exchange of photons [9].

An alternative way to entangle two systems without ever getting into contact can be achieved by entanglement swapping. Two photons emitted by the two atoms can be jointly measured [10]. The outcome of the measurement defines a local feedback action one has to apply to finish the entangling protocol. This ansatz is avoiding the difficulties that arise when a single photon has to be re-absorbed by a single atom. Even in the case where we restrict to linear optical elements, entanglement can be reliable and efficient generated in a probabilistic way using measurement and local feedback action [10].

Taking advantage of this idea for quantum simulation is hindered by two problems: (i) Using measurements to run a quantum simulation is an unusual concept since this
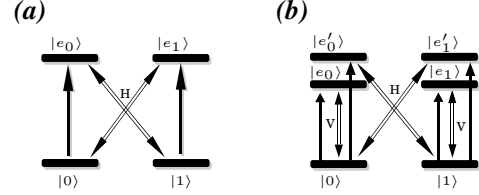

FIG. 1: The atom stores a qubit in two ground states. In addition several excited levels are used to create photons in the cavity. The double arrays indicate photon modes supported by the cavity, whereas the single arrows stand for external laser pulses to excite the atoms. In (a) only one photon mode is supported by the cavity. The photon qubit is encode is the presence or absence of a photon. In (b) two different photon modes are supported.

does not lead to an unitary time evolution. The emitted photons have to be entangled with the atoms such that every measurement collapses the atom state to a random outcome. Thus, every single atom runs through its own random independent evolution and it may be hard to synchronize all the atoms to simulate one controlled unitary evolution. (ii) In a linear optical setup a complete Bell-measurement, as required for entanglement swapping, is not available [8]. In consequence, the desired interaction can only be generated in a non-deterministic way. In addition, a realistic photon is highly vulnerable to absorption and measurement losses.

In this letter we propose a scheme to circumvent those problems and to simulate effectively arbitrary next neighbor interactions among individually distance atoms with the help of emitted photons and measurements. Although each individual evolution is random, the system can be forced efficiently to follow any specific unitary next-neighbor evolution. The scheme can also be made resistant against any photon loss rates by keeping a backup copy of each photon.

Photon generation: We consider atoms each in an optical cavity with the two ground states $|0\rangle$ and $|1\rangle$ realizing a qubit [7]. We assume that we can reliably apply any local unitary on the level structure of a single atom via lasers and external electromagnetic fields. To couple two

atoms we need the atoms to emit photons. For this purpose we need to include more internal levels of the atoms. We assume here two different possibilities depending on whether we want to encode the photon-qubit into two different polarizations or the photon occupation number (photon/ no photon) :

(a) *Photon/no photon:* The atom has the two excited levels $|e_0\rangle$ and $|e_1\rangle$, which decay via the same cavity mode into the ground state as shown in Figure 1 (a). The two ground states can simultaneously be changed in the two excited state by a laser pulse. By changing the strength of the laser pulse we can bring the ground states $|0\rangle, |1\rangle$ into any superposition of the ground and the excited states, i.e., $|0\rangle \rightarrow \sqrt{1-\varepsilon}|0\rangle + \sqrt{\varepsilon}|e_0\rangle$ and $|1\rangle \rightarrow \sqrt{1-\varepsilon}|1\rangle + \sqrt{\varepsilon}|e_1\rangle$. When the excited state decays the atom emits a photon such that we get the transformation

$$
\begin{aligned}
|0\rangle &\rightarrow \sqrt{1-\varepsilon}|0\rangle|V\rangle + \sqrt{\varepsilon}|1\rangle|H\rangle, \\
|1\rangle &\rightarrow \sqrt{1-\varepsilon}|1\rangle|V\rangle + \sqrt{\varepsilon}|0\rangle|H\rangle
\end{aligned}
\tag{1}
$$

where $|H\rangle$ denotes a photon in the cavity mode, while $|V\rangle$ stands for the vacuum, i.e., the absence of any photon.

(II) *Polarized photons:* A similar kind of transformation can be archived using the level structure showed in Figure 1(b). The atom has four excited levels and the cavity supports two different cavity modes, called H and V. If the atom is in level $|e_0\rangle(|e_1\rangle)$ it decays into $|0\rangle$ $(|1\rangle)$ and emits a V-photon to the cavity, while when in state $|e_0'\rangle$ $(|e_1'\rangle)$ it decays to $|1\rangle$ $(|0\rangle)$ and emits a H-photon. With two laser pulses we can excite the ground states into a superposition of the four excited states $|0\rangle \rightarrow \sqrt{1-\varepsilon}|e_0\rangle + \sqrt{\varepsilon}|e_0'\rangle$ and $|1\rangle \rightarrow \sqrt{1-\varepsilon}|e_1\rangle + \sqrt{\varepsilon}|e_1'\rangle$, where the ε can be controlled by the strength of the lasers. After the atom has decayed back into the ground states we get a transformation similar to (1) with the only difference that $|V\rangle$ stands now for a photon and not for the vacuum.

In both cases the original qubit encoded in the ground state gets flipped with probability ε where the flip can be recognized by a measurement of the emitted photon. This basic operation will be denoted as U_ε.

Feedback loop: To get an effective interaction between two atoms we chose the following strategy: we apply U_ε to the first atom and $U_{(1-\varepsilon)}$ followed by a flip F to the second atom. The flip operations exchanges the states $|0\rangle$ and $|1\rangle$. The two (possible) photons emitted by the two atoms are sent onto a beam-splitter and measured. At one branch we add an extra phase of i to the H-photon (see Fig. 2).

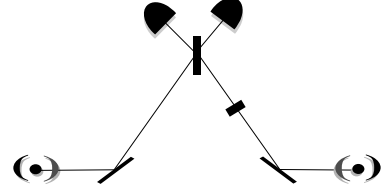If $|\psi\rangle$ denotes the initial state of the two atoms we will

FIG. 2: Two distance atoms are effectively couplet by sending photons coming out of the cavity to a beam splitter. Dependent on the measurement outcome we apply a feedback loop on the atoms until the desired evolution has been done.

get after emitting the photons the state

$$
\begin{aligned}
U_\varepsilon \otimes F U_{(1-\varepsilon)}|\psi\rangle =& \\
(1-\varepsilon)|\psi\rangle|VH\rangle& \\
+i\varepsilon\sigma \otimes \sigma|\psi\rangle|HV\rangle& \\
+\sqrt{\varepsilon(1-\varepsilon)}1 \otimes \sigma|\psi\rangle|VV\rangle& \\
+i\sqrt{\varepsilon(1-\varepsilon)}\sigma \otimes 1|\psi\rangle|HH\rangle,&
\end{aligned}
\tag{2}
$$

where σ is the first pauli matrix that flips the two states $|0\rangle, |1\rangle$. After passing the beam-splitter the photons are measured in the H, V basis. This correspond to an incomplete Bell-measurement of the two-photon pair, i.e., they are projected in one of the four states $|-\rangle := |HV - VH\rangle, |+\rangle := |HV + VH\rangle, |HH\rangle, |VV\rangle$. The remaining two atoms are projected onto

(i) $|-\rangle \mapsto (1-\varepsilon)|\psi\rangle - i\varepsilon\sigma\otimes\sigma|\psi\rangle$ with probability $\frac{1}{2}((1-\varepsilon)^2 + \varepsilon^2)$;

(ii) $|+\rangle \mapsto (1-\varepsilon)|\psi\rangle + i\varepsilon\sigma_\otimes\sigma|\psi\rangle$ with probability $\frac{1}{2}((1-\varepsilon)^2 + \varepsilon^2)$;

(iii) $|VV\rangle \mapsto (1 \otimes \sigma)|\psi\rangle$ with probability $\varepsilon(1 - \varepsilon)$;

(iv) $|HH\rangle \mapsto (\sigma \otimes 1)|\psi\rangle$ with probability $\varepsilon(1 - \varepsilon)$.

Let us first assume that the only possible outcomes would be $|+\rangle, |-\rangle$. In both cases the resulting state can be written as $e^{it\sigma\otimes\sigma}|\psi\rangle = \cos(t)|\psi\rangle + i\sin(t)\sigma \otimes \sigma|\psi\rangle$ with t equal to $t = \pm\arcsin(\varepsilon) \approx \varepsilon$. So we can simulate any time evolution with respect to the Hamiltonian $\sigma\otimes\sigma$ over a time $t = \pm\varepsilon$, but with a random time direction, i.e., by repeating we can simulate a random walk on the time axis of the evolution. If we aim for a time simulation $V(t) = e^{it\sigma\otimes\sigma}$ we can repeat the protocol until the random walk by chance matches the desired time t. We can speedup this strategy by a feedback loop controlling the ε parameter. In a first step we choose ε in such a way, that $\sin t = \varepsilon$ giving us a chance of $1/2$ to succeed with $V(t)$ in the first step. We will assume t to be small such that we can approximate $\varepsilon = t$ in the following discussion, but in principle everything can be done exactly. In a case of a failure $|-\rangle$ we end up with $V(-t)$. By increasing ε to 2ε we apply randomly $V(\pm 2t)$ in the next

step such that in the successful $|+\rangle$-case we compensate the previous $V(-t)$ rotation and in addition are left with the desired $V(t)$ rotation. In case of further failures we each time double ε and retry. This way we guarantee for a constant success probability of $1/2$ in each round and we can simulate every evolution $V(t)$ with exponential increasing probability in the number of rounds.

Problems may occur if we measure $|HH\rangle, |VV\rangle$ in between. But in these cases we applied either $1\otimes\sigma$ or $\sigma\otimes 1$ which can be corrected by local gates. Since our Hamiltonian $\sigma\otimes\sigma$ commutes with these unitaries we can ignore these measurement results and just correct the overall error in the end. Note that this overall error only can be one of the three local unitaries $1\otimes\sigma, \sigma\otimes 1, \sigma\otimes\sigma$. Since the probability of $|+\rangle, |-\rangle$ is at least $1/4$ in each round the success probability is still exponentially good. So we can efficiently simulate $V(t)$ for arbitrarily t up to some random but known Pauli-errors that commute with the evolution.

Simulations for $H = \sigma_k \otimes \sigma_l$: Given $V(t)$ and the possibility to do local unitary gates at each single atom, we can apply

$$V^{kl}(t) = e^{it\sigma_k\otimes\sigma_l} \approx (1 + it\sigma_k \otimes \sigma_l) \quad (3)$$

for arbitrarily Pauli matrices $\sigma_k \otimes \sigma_l$, since $V^{kl}(t) = u_k \otimes u_l V(t) u_k^\dagger \otimes u_l^\dagger$ with the local unitaries defined by $u_k = \sigma_k^{\frac{1}{2}}\sigma_1^{-\frac{1}{2}}$. If the measurement outcome is $|HH\rangle, |VV\rangle$ we pick up an error of $1\otimes\sigma_l$ or $\sigma_k\otimes 1$ which commutes with (3).

Simulation of an arbitrarily two-qubit Hamiltonian: Every two-qubit Hamiltonian can be written as $H = \sum_{kl}\lambda_{kl}\sigma_k\otimes\sigma_l$. By successively applying $V^{kl}(\varepsilon\lambda_{kl})$ we can simulate

$$\prod_{kl} V^{kl}(\varepsilon\lambda_{kl}) = 1 + i\varepsilon\sum_{kl}\lambda_{kl}\sigma_k\otimes\sigma_l + O(\varepsilon^2)$$

which equals $e^{i\varepsilon H}$ up to first order in ε. Note that each $V^{kl}(\varepsilon\lambda_{kl})$ can produce an error that does not commute with the following $V^{k'l'}(\varepsilon\lambda_{k'l'})$. But since only products of Pauli-matrices are involved, the possible errors either commute or anti-commute with the following evolutions. In the case that $V^{k'l'}$ anti-commute only the time direction is inverted which we easily can compensate by exchanging the $|+\rangle$ and the $|-\rangle$ case in above protocol. So we can simulate any product $\prod_{kl} V^{kl}$ up to a random but known error $\sigma_r\otimes\sigma_p$, i.e., $|\psi\rangle \rightarrow \sigma_r\otimes\sigma_p\prod_{kl}V^{kl}|\psi\rangle$. This allows us to take advantage of the Trotter-formula [11] to approximate e^{itH} by

$$\lim_{n\to\infty}\left[\prod_{kl}V^{kl}\left(\frac{t}{n}\lambda_{kl}\right)\right]^n = e^{itH}$$
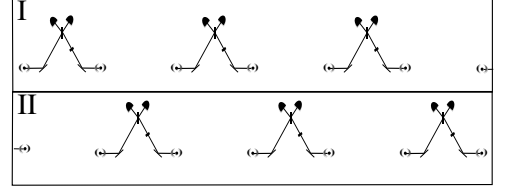
with a precession of the order of $\frac{1}{n}$.

FIG. 3: To simulate a next neighbor Hamiltonian we can do $m/2$ of the rotations in parallel. We need to basic setup such that each cavity sends a photon on a beam splitter together with either its right or its left neighbor.

Simulation of an arbitrarily next neighbor Hamiltonian: In the same manner we can simulate any next-neighbor Hamiltonian on a lattice, e.g. the 1D Hamiltonian $H = \sum_{kl,x=1}^{x=m}\lambda_{kl}^{(x)}\sigma_k^{(x)}\otimes\sigma_l^{(x+1)}$, where x labels the different qubits. Using the Trotter-formula and the V^{kl} we can approximate e^{itH} by

$$\lim_{n\to\infty}\left[\prod_{klx}V_x^{kl}\left(\frac{t}{n}\lambda_{kl}\right)\right]^n = e^{itH}$$

with a precession of the order of $\frac{m}{n}$ up to an overall error that is just a product of random but known Pauli-matrices. The same approximation can be made for any lattices with next-neighbor interactions or for any arbitrary Hamiltonian consisting of sums of two-qubit Hamiltonians.

Time estimation: Assume we want to simulate a Hamiltonian $H = \sum_{i=1}^{m'}H_i$, where H_i denotes a two qubit Hamiltonian. Using the Trotter formula we can approximate the time evolution e^{iHt} by applying $4mn$ V^{kl}-rotations. To get a constant precision we have to scale n at least linearly with m such that we need $O(m^2)$ applications of a single V^{kl}-rotation for a proper simulation. For every V^{kl}-rotation we need one successful application of the feedback loop which succeeds with a probability of about $\frac{1}{2}$ (or at least $\frac{1}{4}$). If we apply $2m^2 + \frac{const}{2}m$ steps we get at least m^2 successfully feedback loops with a probability that exponentially approaches 100%, e.g. 97% for $const = 3$. Therefore we can simulate any Hamiltonian with m two-qubit terms in a time that scales quadratically with m.

In a $1D$ m-particle next neighbor setups the Hamiltonian will consist out of m terms. Since only two qubits are involved in each local term we can speed up the simulation by applying at a time $m/2$ of the steps in parallel (see Fig. 3). Assume we want to make $m/2$ gates in parallel in a time that allows for $c\log(m)$ feedback loops. Then each single gate succeeds with $(1 - 2^{-c\log(m)}) = (1 - m^{-c\log(2)})$. So the total probability of success of the $m/2$ gates is $(1 - m^{-c\log(2)})^{m/2}$ which approaches 1 as long as $c\log(2) > 1$. This whole procedure has to be repeated $2m$ times to do the required m^2 V^{kl}-rotations. Thus, we can simulate with arbitrar-

ily high probability m particles in a 1D setup in a time scaling like $O(m \log(m))$.

Errors: The protocol proposed so far is highly vulnerable to photon losses and detection losses. A photon loss is especially fatal if we miss to identify a $|HH\rangle$ or $|VV\rangle$ measurement result, because we miss to identify a Pauli-error.

In the case we work with polarized H and V photons we know whether we missed to detect a photon or not, since there are always two photons generated. We can use this fact to fight photon losses by the following idea. We will keep a backup-copy of the photon that we can use in the case that the real photon is lost. To store this copy assume a second 'atom' is present in each cavity. So we have two atoms (called A and B atom) with two internal levels each and we assume that we can apply arbitrarily two qubit gates between them. In the first step the B-atoms will play now the role of the photons to get the backup-copy before the photon is even generated: We start with the two atom state $|\psi\rangle_{AA'}$ and the two extra atoms in state $|00\rangle_{BB'}$ and apply U_ε now between the two atoms A and B (and A' and B'), i.e., $|0\rangle_A|0\rangle_B \to \sqrt{1-\varepsilon}|0\rangle_A|0\rangle_B + \sqrt{\varepsilon}|1\rangle_A|1\rangle_B$ and $|1\rangle_A|0\rangle_B \to \sqrt{1-\varepsilon}|1\rangle_A|0\rangle_B + \sqrt{\varepsilon}|0\rangle_A|1\rangle_B$. After applying this we are left with a state of form (2) where the photon modes V, H are replaced by the internal states $|0\rangle, |1\rangle$ of the two B-atoms. The next step is to copy the internal states of the B-atoms onto photons, such that we exactly get (2) and can continue the protocol.

To this end we have to force the two B-atoms to produce a photon dependent on their internal level, i.e., $|0\rangle_B \to |0\rangle_B|V\rangle, |1\rangle_B \to |1\rangle_B|H\rangle$. Note that the B-atom will keep their quantum information and can be reused in the case of a photon loss. If we measure both B-atoms in the $|0+1\rangle, |0-1\rangle$ basis then we destroy this information and are left with a state similar to (2) up to some random but known sign-phases that will change only the time directions. So we can use the above protocols up to the fact, that we have to include the measurement results of the B-atoms to identify the direction of time evolution.

Instead of measure the 'B' atoms in the $|0+1\rangle, |0-1\rangle$ basis we can measure them individually in the standard basis $|0\rangle, |1\rangle$. In this case $|\psi\rangle$ stays unchanged up to one of the random but known errors $1 \otimes \sigma_l, \sigma_k \otimes 1, \sigma_k \otimes \sigma_l$. We do not get any time evolution, but are still able to continue the protocol.

The main idea is now to first measure the photons and then the atoms. If both photons arrived we continue by measuring the B-atoms in the $|0+1\rangle, |0-1\rangle$ basis. If one or both photons are missing we measure in the $|0\rangle, |1\rangle$ basis to protect our system against random unknown errors. Note that this measurement can be repeated until we successfully detected a photon. Doing so, we can handle any photon loss rates.

Instead of putting two atoms in a cavity we can as well use one single atom with four internal levels. In the four
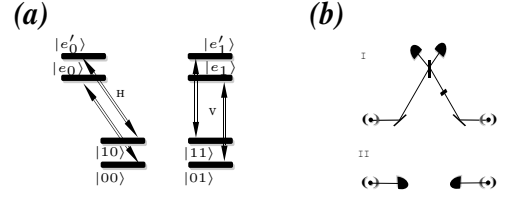

FIG. 4: (a) to fight photon losses the atom needs to store a second qubit to carry a security copy of the photon. (b) in the first attempt we try to establish an evolution by sending the photons on a beam-splitter. In case of success we erase the security copy, in case of failure we can reuse the copy by measuring in different basis.

internal levels we can encode the A and the B atom. To this end we need an internal level structure that allows for storing two-qubits and the possibility to measure one of this qubits independently from the other. On possible level structure is shown in Fig. 4. The cavity supports two photon modes. If we excite $|10\rangle, |00\rangle$ to $|e_0'\rangle, |e_0\rangle$ and $|11\rangle, |01\rangle$ to $|e_1'\rangle, |e_1\rangle$, we can measure the second qubit while the first qubit stays untouched.

Quantum computation: Note that the same kind of setup can be used to do a quantum computation. If we apply $V(t)$ to two atoms with t equal to $\pi/2$ we get a two-qubit gate that is locally equivalent to a c-not gate. Together with local gates on single atoms this allows for universal quantum computation. This scheme is similar to the so called 'repeat until success'-schemes proposal in [12]. The advantage we have here is that in our proposal a single two-qubit gate can be made photon loss resistant, whereas in [12] photon losses can only be attacked via the creating of cluster states which then are used as a resource for one way quantum computation.

Conclusion: We have proposed a scheme for performing quantum simulations based on a photon detection feedback loop that requires only linear optical elements. Atoms can be stored individually without the need to directly realize an interaction between two atoms. The scheme can simulate any time evolution that can be written as a sum of two-qubit Hamiltonian in a time scaling polynomially with the number of particles. Our scheme can be modified to be resistant to any amount of photon losses by storing two qubit per cavity, whereas the second qubit carries a backup-copy of the photons. The same kind of setup can be used to do c-not gates and implement a universal quantum computer. So far the simulation schemes needs several discrete round. One challenge would be to make the interaction and the feedback control continuous [13] and establish a continuous feedback theory for a quantum simulator. We acknowledge support from DFG-Forschungsgruppe 635 and SCALA.

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